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A modified image watermarking method with compressed watermark based on DCT-SVD transforms

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Abstract

In this work, an enhanced watermarking method is proposed. The SVD transform is utilized to compress the watermark image, so the capacity, imperceptibility, and security are increased. The cover image is separated into blocks, and the embedding is carried out on the highest variance blocks to increase the imperceptibility of the method. The selected blocks are transformed using the Discrete Cosine Transform (DCT), a matrix including the DC coefficients is created, and transformed by the Singular Value Decomposition (SVD). The compressed watermark's principal components are embedded into the singular values of the cover image. A robust method is obtained by using the DCT-SVD transforms. A watermark with different sizes can be embedded without association with the number of image blocks, while the size of the watermark in a similar method is restricted by the number of cover image blocks in rows and columns. High Peak Signal to Noise Ratio (PSNR) values are demonstrated by experimental results lying in the range (45.5-48.5) for the watermarked images of size 512*512 and a watermark of size 64*64, and it is possible to extract an acceptable quality watermark from the attacked image.

1. Introduction

The Internet and technology are advancing rapidly, so multimedia information is propagated and shared easily [1]. Copyright protection of the multimedia has been of considerable interest during the past few decades, and the problem of quickly and effectively securing the copyright has emerged. Digital watermarking has been deemed a proper solution for safeguarding the copyright of multimedia content [2]. The image that has a watermark inserted in it is named a watermarked image; the watermark can be extracted by the owner to show their ownership if the watermarked image is shared [3, 4].

The watermarking techniques have a considerable role in securing digital images. The techniques hide copyright signs in host images robustly and imperceptibly. [5]. The watermark embedding and extracting are the pivotal stages in any watermarking technique [6]. In the embedding stage, the watermark is incorporated into the host image by employing the watermarking embedding technique. After transmitting the watermarked image through the network, which may be affected by attacks, the watermark is extracted by using the watermark extraction technique [7]. Watermarking techniques can be categorized into three sets depending on the extraction techniques as follows: blind, semi blind, and non-blind. In semi-blind, the extraction step requires the keys and the watermark. The host and the watermark images are needed for non-blind techniques. In the blind technique, the original keys are needed for extraction [6].

The watermarking approaches are applicable in frequency and spatial spaces [8]. The spatial space utilizes the truth that the modification of the least significant bit (LSB) of data is less sensitive to the human visual system, so these bits of some pixels in the host image can be considered for embedding [9]. The spatial techniques have a disadvantage in that the embedding capacity is small. When enlarging the capacity, the quality of the watermarked image is degraded. The frequency domain can be used to overcome this disadvantage [10]. The frequency transform is applied on the host image to convert it to a signal space then the watermark is embedded. The inverse transformation is used to extract the watermark [7].

The characteristics of designing a watermarking algorithm are Imperceptibility, robustness, capacity, and security. These characteristics cannot be held concurrently [7]. The two main properties are robustness and imperceptibility. The first means that the watermark can withstand image-processing attacks, while the second means that the host image conceals the watermark invisibly. These two characteristics need to be balanced in designing the watermarking techniques [11].

This work aims to obtain a high-capacity, secure, robust, and imperceptible method based on the Discrete Cosine Transform (DCT) and Singular Value Decomposition (SVD). The watermark image is compressed before embedding. The embedding is performed on the DC coefficients of the selected blocks after transforming them by the SVD. The highest variance blocks are selected for embedding. The highest variance blocks represent a complicated image texture; these blocks are less sensitive to the human eye [12, 13]. Different watermark sizes can be embedded by the proposed method, and the number of selected blocks relies on the size of the compressed watermark, unlike similar methods such as [14], which can embed a watermark with a size restricted to the number of cover image blocks in rows and columns.

2. Related works

In N. Muhammad et al. [1], the watermark image was decomposed using partial pivoting lower and upper triangular (PPLU) decomposition. The upper and lower production matrices were embedded. The cover image is transformed into subbands by the wavelet transform then the singular value decomposition is applied. The scaling factor is determined by utilizing the differential evolution technique, which can give the best robustness against pirate attacks and image processing. Y. AL-Nabhani et al. [15] suggested an algorithm that is invisible and could extract the watermark without the existence of the source image. The discrete wavelet transform is utilized to embed the binary image in specified coefficient blocks, and the watermark is extracted using a probabilistic neural network. S. Liu et al. [16] proposed a technique that enhances the traditional Discrete Cosine transform (DCT) watermarking technique by combining fractal encoding with it to obtain double encryption. J. Liu [17] proposed a DWT, singular value decomposition (SVD), Hessenberg decomposition (HD) based technique that utilizes the fruit fly optimization method to specify the scaling factor. R. Singh et al. [14] proposed an imperceptible and robust technique using the DCT and SVD transform simultaneously. The DCT transform is widely used in many multimedia applications, including image watermarking, due to its excellent energy compaction property; however, it is vulnerable to attacks and is therefore combined with the SVD transform, which is valuable for extracting geometric multimedia information. R. Singh et al. [6] proposed a technique that overcomes the false positive problem (FPP). This problem gave access to the multimedia to the wrong owner. The technique uses the Arnold transform to shuffle the host and watermark images, followed by DWT and SVD. M. Begum et al. [7] suggested a hybrid technique that encrypts the watermark by using the Arnold method, and then applies DCT, DWT, and SVD to both the watermark and host images. H. K. Singh et al. [5] employed a convolutional neural network (CNN) to develop a digital watermarking technique. An encoder network is used to extract the latent feature of the host and watermark images. At the receiver, a denoising autoencoder network is used to remove noise from the image, and a CNN is then used to extract the watermark.

W. Wu et al. [18] proposed a robust method that employs DWT and SVD. The logistic-tent map, in combination with the Arnold transform are used to encrypt the watermark. The host image is transformed using the DWT, and the SVD is then applied to its high-frequency subband. The singular values of the watermark are embedded in the singular values of the host image. M. F. Saikat et al. [19] suggested a method that uses the YIQ colour space and deep learning. The embedding and extracting phases are improved by combining the Catalan transform (CT) and non-negative matrix factorization (NMF) with an Artificial Neural Network (ANN). S. K. Ahmed [20] proposed a method based on the DCT transform that utilizes the relationship between two DCT coefficients. The transformed blocks are separated into two groups: the first corresponds to the greater-than relation, and the second to the less-than relation. The watermark pixels with value zero are embedded in the first group, and the second group for the value one. The embedding is performed by keeping the address of the specified blocks. S. Syafiqul et al [21] presented a method that uses IWT, DCT, and SVD. The cover is processed by an R-level IWT transform, yielding (HH, LH, HL, LL) subbands. The 8*8 DCT transform further processes the LL sub-band, and the resulting coefficients are transformed by the SVD. The singular values of the watermark image are embedded in the singular values obtained from the cover. The watermarked image is gained by applying the inverse transforms of the SVD, DCT, and R-level IWT.

In many watermarking techniques, the watermark size is limited; it depends on the number of available elements in the cover image, such as the number of image blocks, the number of singular values, and others. Also, as the watermark size increases, the imperceptibility decreases. The proposed method overcomes these limitations by compressing the watermark and embedding it in blocks with the highest variance.

3. Theoretical background

3.1 Discrete cosine transform

DCT is a transform that can be utilized to translate the spatial domain image to the frequency domain and is commonly applied in image watermarking methods [22]. Usually, the image is split into 8*8 subblocks prior to being converted by DCT. The transformation of the block generates 64 components [23]. One component, named DC, stands for the block's colour average, and the other 63 components represent the alteration of the colour inside the blocks [24]. The DCT for the image f of size M*N can be calculated using Equation (1). The transformed image is denoted by f^{-1} [22, 25].

$$f^{-1}(u,v) = \propto(u) \propto(v) \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) \cos\frac{(2m+1)u\pi}{2M} \cos\frac{(2n+1)v\pi}{2N} \quad (1)$$

where

$$\propto(u) = \begin{cases} \sqrt{\frac{1}{M}} & u = 0 \\ \sqrt{\frac{2}{M}} & otherwise \end{cases} \quad (2)$$

$$\propto(\mathrm{v}) = \begin{cases} \sqrt{\frac{1}{N}} & v = 0 \\ \sqrt{\frac{2}{N}} & otherwise \end{cases} \quad (3)$$

The transformed matrix f^{-1}can be converted to the original matrix f using Equation (4) [22], [25]

$$f(u,v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \propto(u) \propto(v) f^{-1}(m,n) \cos\frac{(2m+1)u\pi}{2M} \cos\frac{(2n+1)v\pi}{2N} \quad (4)$$

3.2 Singular value decomposition

The symmetric matrix can be partitioned using the SVD method into three submatrices so that the singular values are collected in a diagonal matrix [17]. An image X, with dimension M*N, M>=N, represented by the SVD as Equation (5).

$$[X] = [U][S][V]^T \quad (5)$$

Where U_{M*M} and V_{N*N} are orthogonal matrices whose columns are called left singular vectors for the U matrix and right singular vectors for the V matrix. The S_{M*N} matrix is a diagonal matrix with values representing the singular values of X [26], [27]. The columns of the matrix U represent the left eigenvectors of XX^T. The columns of V are the right eigenvectors of X^TX. The diagonal matrix S contains the square roots of the eigenvalues in descending order from the U and V matrices [26], [27]. Assume the rank of X is r(r<n). The diagonal elements of the matrix S must satisfy the Equation (6). The image layer luminance is specified by each singular value, while the geometry of the image is specified by the corresponding pairs of singular vectors [28].

61>= 62>= ………>= 6r>= 6r+1= 6r+2= 6r+3…….= 6n=0 (6)

The SVD transform is used widely for compressing the image because of its ability to find a new image representation with a lower rank [29]. The compression is done by ignoring some small singular values with the corresponding singular vectors [30].

4. Methodology

In this work, a modified technique is achieved with high capacity, imperceptibility, robustness, and security. This improvement is reached by compressing the watermark using the SVD compression before embedding, so the capacity is increased. Also, the embedded blocks are selected in such a way that minimizes the effect of embedding on the quality of the watermarked image. The highest variance blocks are selected for embedding because they have little effect on the quality of viewing the image in human eyes. A high imperceptibility and security are achieved better than embedding the watermark in a sequential block. The watermarking method is applied in two phases as follows:

4.1 Embedding phase

In this phase, the watermark is embedded in the cover image to obtain the watermarked image. The embedding steps are as follows:

1- The cover image of size (x*y) is partitioned into 8*8 blocks. The variance for each block is computed and stored in a list VAR of size x/8*y/8.

2- The VAR list is sorted in descending order.

3-The watermark image is divided into 8*8 blocks. Each block is compressed using the SVD transform, which partitions the block into Ui,j, Si,j, and Vi,j. Where i,j stand for the row and column number of the block.

4- Compress each block of the watermark by keeping Ki,j singular values and singular vectors. Construct a list KSVD which contains the value of K for each block.

5- Construct a matrix PC (8 rows by m columns, where m represents the number of retained pc from all blocks) which contains the principal components for all blocks. The principal component for a block is computed as follows:

b_pci,j=U(:,1: Ki,j)* Si,j(1: Ki,j, 1: Ki,j) (7)

6- Arrange a matrix PC as a square matrix.

7- Create a matrix VV which contains the retained Vi,j for all blocks. This is used as a key in the extracting phase.

8- Select a number of blocks from the cover image equal to the number of elements in the PC matrix. The blocks are selected starting from the highest variance block and continue until the required number of blocks.

9- Apply DCT on the selected blocks

10- Construct a matrix DC that contains the DC components of the transformed blocks

11- The DC matrix is transformed by the SVD transform:

$$DC=USV^T \quad (8)$$

12- Add the PC to the diagonal matrix S with embedding factor α:

$$Si=S+\alpha PC \quad (9)$$

13- The Si matrix is transformed by the SVD, so we obtain Uc , Sc ,and Vc.

$$Si=U_cS_cV_c^T \quad (10)$$

14- Create the AW matrix as follows:

$$AW=U\,S_c\,V^T \quad (11)$$

15- Change the DC values of the selected blocks with the associated AW values.

16- The watermarked image is obtained by performing the IDCT on the selected blocks.

4.2 Extracting phase

The watermark is extracted from the watermarked image at this phase. The steps are as follows:

1- The watermarked image is separated into 8*8blocks.
2- The DCT is applied to the selected blocks.
3- The DC values from the transformed blocks are collected in a square matrix DCW.
4- The DCW matrix is transformed by the SVD.

$$DCW=U_W S_W V_W^T \tag{12}$$

5- Find the extracted matrix Si*.

$$Si^*=U_c S_W V_c^T \tag{13}$$

6- Extract the PC* matrix:

$$PC^*=(Si^*-S)/\alpha \tag{14}$$

7- Arrange the matrix PC* as 8 rows by m columns.
8- Extract the watermark blocks by performing the ISVD. For each block(i,j), the number of columns taken from PC* and VV matrices is Ki,j, which is obtained from the matrix KSVD.
9- Merge the extracted blocks to obtain the watermark.

The proposed method is semi-blind. In the extracting phase, some auxiliary information obtained from the embedding phase is required, including the embedding blocks' position, embedding factor, the singular value matrix S of the DC matrix, and the singular vectors U_c *and* V_c.

5. Experimental results

The proposed method is applied to five gray images of size 512*512 Fig.1 (a-e). The embedding factor α is 0.3, and the watermark is compressed using 2% discarding percentage of the sum of the singular values with the corresponding singular vectors. The imperceptibility and robustness of the suggested method are evaluated.

The Peak Signal to Noise Ratio (PSNR) is utilized to measure the quality of the watermarked image, and the structural similarity index measure (SSIM). These metrics are used for imperceptibility evaluation. Equation (15) represents the PSNR.

$$PSNR = 10\, log_{10} \frac{(255)^2}{MSE}\, dB \tag{15}$$

$$MSE = \frac{1}{m*n} \sum_{i=1}^{n} \sum_{j=1}^{m} [A(i,j) - AW(i,j)]^2 \tag{16}$$

Where A represents the host image, and AW is the watermarked image.
The SSIM(A, AW) is calculated as follows:

$$SSIM(A, AW) = \frac{(2\mu_A \mu_{AW} + C_1)(2\sigma_{AAW} + C_2)}{(\mu_A^2 + \mu_{AW}^2 + C_1)(\sigma_A^2 + \sigma_{AW}^2 + C_2)} \tag{17}$$

The Averages are μ_A and μ_AW, the variances are σ_A and σ_AW, the Covariance is σ_AAW, for the matrices A and AW, respectively.

The division is stabilized with a weak denominator using the variables C1 and C2 [17].

The robustness of the extracted watermark is measured using Normalized Correlation (NC) according to Equation (18).

$$NC = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} A(i,j) * AW(i,j)}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} A(i,j)^2}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} AW(i,j)^2}} \quad (18)$$

a b c d e

Fig. 1. Host images (a) Man (b) cameraman (c) Boat (d) Baboon (e) Peppers.

In Fig. 2, different watermark sizes are compressed and embedded in the pepper image of size 512*512. The watermarks size are 32*32, 64*64, and 128*128. Without compressing the watermark, the maximum watermark size that can be embedded in 512*512 cover image is 64*64, while the larger size can be embedded after compressing it, as proposed in this work. It can be seen from Fig. 2 that the PSNR decreases as the watermark size increases because more information is embedded.

Original Watermark	 Size 32*32	 Size 64*64	 Size 128*128
Compressed Watermark	 NC = 0.9999 PSNR = 40.1890	 NC = 1.0000 PSNR = 43.8369	 NC= 1.0000 PSNR_w = 46.1439
Watermarked Image	 PSNR = 51.4928 ssimm = 0.9985	 PSNR = 48.4899 ssimm = 0.9950	 PSNR = 42.9064 ssimm = 0.9778

Fig. 2. The embedding of different size compressed watermark in the Pepper image of size 512*512.

5.1 imperceptibility evaluation

The imperceptibility expresses the likeness between the watermarked and the host image. The high imperceptibility means a little difference between them and vice versa [14]. Fig. 3

illustrates the objective and subjective quality of the watermarked images for 3 cases. The first embeds the watermark directly without compression, and the embedding blocks are sequential. The second uses a compressed watermark and sequential embedding blocks. The third uses a compressed watermark, and the embedding blocks are specified based on the variance. The results show that the compressed watermark gives better results than the uncompressed watermark, and specifying the highest variance blocks for embedding is better than sequential embedding blocks. High PSNR and SSIM values are obtained, which indicate a highly imperceptible system.

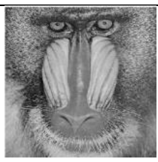
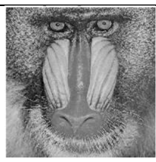
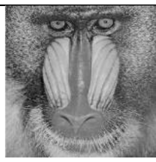
Watermark image of size 64*64			
Image Name	Watermarked Image without Compression, sequential embedding blocks	Watermarked Image with Compression, sequential embedding blocks	Watermarked Image with Compression, high variance embedding blocks
Peppers	 PSNR=45.7745 ssimm = 0.9910	 PSNR = 46.4178 ssimm = 0.9886	 PSNR = 48.5394 ssimm = 0.9953
Man	 PSNR = 45.6891 ssimm = 0.9962	 PSNR = 45.9115 ssimm = 0.9923	 PSNR = 47.7884 ssimm = 0.9961
Boat	 PSNR = 44.0957 ssimm = 0.9965	 PSNR = 45.5821 ssimm = 0.9965	 PSNR = 48.1007 ssimm = 0.9976
Baboon	 PSNR = 43.8749, ssimm = 0.9990	 PSNR = 44.4628 ssimm = 0.9988	 PSNR = 45.4786 ssimm = 0.9995
Cameraman	 PSNR = 44.6960 ssimm = 0.9696	 PSNR = 45.5730 ssimm = 0.9592	 PSNR = 48.4851 ssimm = 0.9931

Fig. 3. A comparison between the methods that use compressed, uncompressed watermark, sequential embedding blocks, and specified blocks depending on the highest variance. Image size 512*512, watermark 64*64.

A comparison can be made with other works, for the cover size is 512*512, and the watermark size is 64*64. In [14], which uses a DCT-SVD transform without compressing the watermark, the obtained PSNR is in the range 37.32-39.12 db. The PSNR for the Man image is 39.12, while in this work it is 47.788. In [21], which uses IWT, DCT, and SVD, the PSNR for the boat is 38.3847 and for the pepper image is 38.2632, while in this work are 48.1007 and 48.5394, respectively. In [17], it depends on discrete wave transformation (DWT), Hessenberg decomposition (HD), and singular value decomposition (SVD). The PSNR for the pepper image is 38.217, and in this work is 48.5394.

5.2 Robustness analysis

Robustness means the ability of the method to extract the watermark from the attacked image [17]. To analyze the robustness of the method, the extracted watermark quality from the attached image is assessed. Fig. 4 shows the attacked watermarked images. In spite of these attacks, the watermark can be extracted in an acceptable quality, as shown in Fig. 5. From the results, it can be seen that the extracted watermark is better when the watermark is compressed, and the embedding blocks are selected before embedding.

Gaussian noise	Salt&pepper	Median Filter 3X3	meanFilter
			
Poison noise	Speckle-noise	imsharpen	JPEG compression
			

Fig. 4: The watermarked cameraman images after attacks.

attack	Extracted watermark without compression, sequential embedding blocks	Extracted watermark with compression, highest variance blocks
Gaussian noise(zero mean, 0.001 variance)	 NC = 0.9995	 nc = 0.9998
Salt&pepper (noise density is 0.001)	 NC = 0.9999	 nc = 0.9999
Median Filter 3X3	 NC = 0.9991	 nc = 0.9995
meanFilter	 NC=0.9943	 nc = 0.9973
Poison noise	 NC = 0.9996	 NC = 0.9998
Speckle-noise(variance is 0.001)	 0.9999	 NC= 0.9999
imsharpen	 NC = 0.9985	 NC = 0.9991
JPEG compression (quality is 50)	 NC = 0.9999	 NC= 0.9999
Rescaling 512 → 256 → 512	 NC= 0.9997	 NC=0.9998

Fig. 5. The extracted watermark after attacks.

6. Conclusions

In this work, an improved watermarking technique is achieved by compressing the watermark before embedding, so the amount of embedded information is reduced. The proposed technique achieves less alteration in the cover image, increased embedding capacity, and improved watermark security. The larger watermark can be embedded in the cover image after compressing it, compared with techniques without compression, such as [14]. The embedding blocks are selected in such a way that minimizes the effect of embedding on the quality of the image viewing system. The highest variance blocks provide good selection block results. A high-quality watermarked image is obtained with PSNRs in the range 45.5-48.5 for the watermark of size 64*64 and cover image of size 512*512, which indicates a highly imperceptible system. The watermark is extracted with NC values greater than 0.99 after different attacks, which expresses a highly robust system. The proposed method used constant values of the embedding factor α and a compression discarding percentage, which may not be optimal. In future work, the enhancement can be achieved by using optimization algorithms to specify the optimal values.

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